THE DRAGONS OF DIRECTION DEPENDENT CALIBRATION

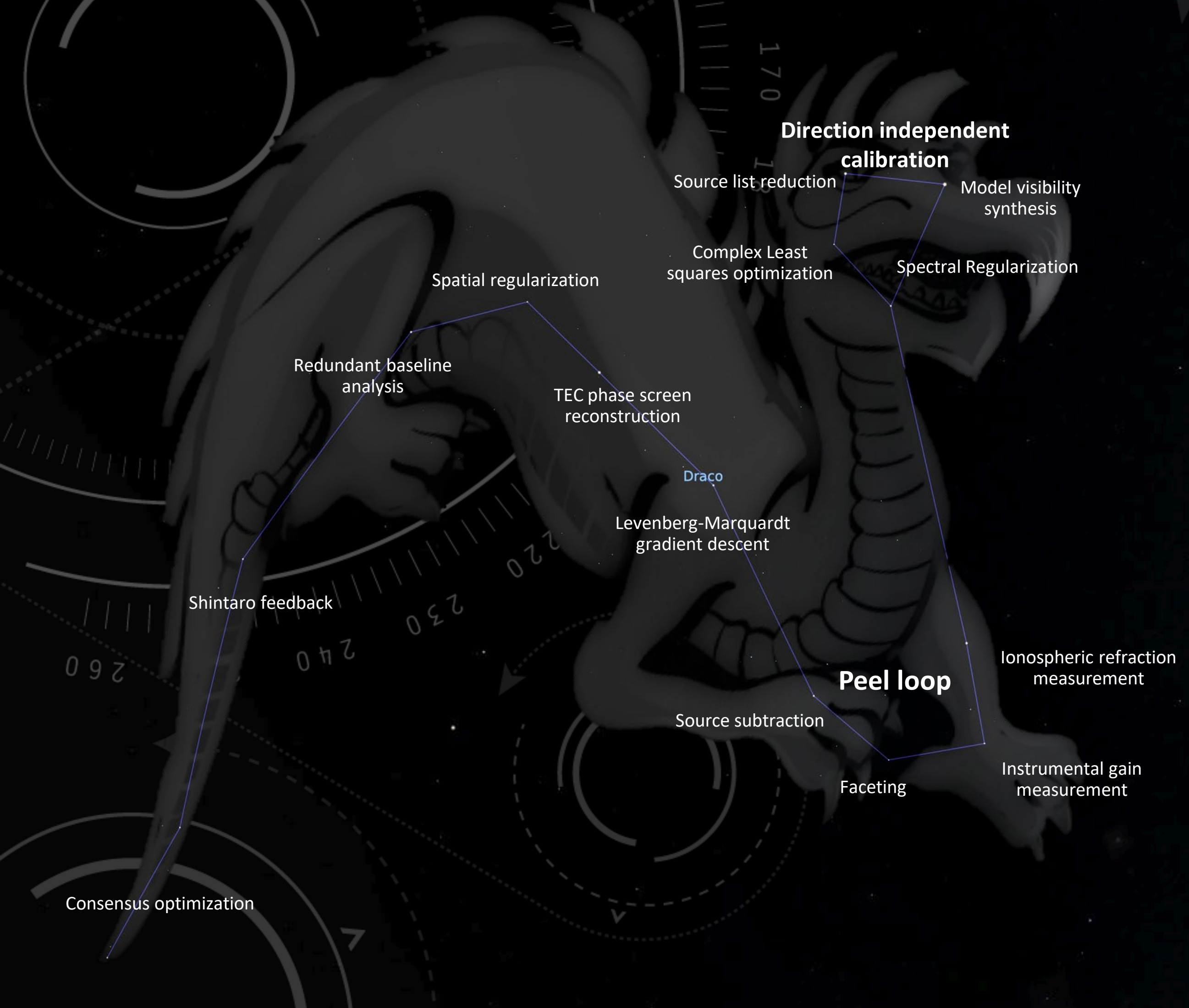


Image credit: Stellarium

Mx Dev Null (they/them) and Dr Chris Jordan (he/him)

MWA Project Meeting 2022-07-21

MOTIVATION

- Understanding calibration is vital for MWA EoR.
- RTS most common for direction dependent calibration
- Optimized to run as fast as possible on exactly 25 GPUs, not optimized for ease of use, flexibility, readability
- No MWAX support, Inflexible, lack of tests, Internals obscured by lack of documentation and non-modular design

- Hyperdrive is portable, robust, extensible, well-documented, well-tested
- Excellent foundation for modular, testable direction dependent calibration with accessible documentation.

CALIBRATION

Raw data contain imprints from the instrument and its environment that need to be corrected:

A. Thermal effects in amplifiers

B. Beam response

C. Cable reflections at kinks or impedance changes

E. Electromagnetic properties of antenna and environment: moisture level, temperature

F. Faraday rotation

I. Ionospheric refraction

P. Parallactic rotation

U. Other, un-modelled effects / interference

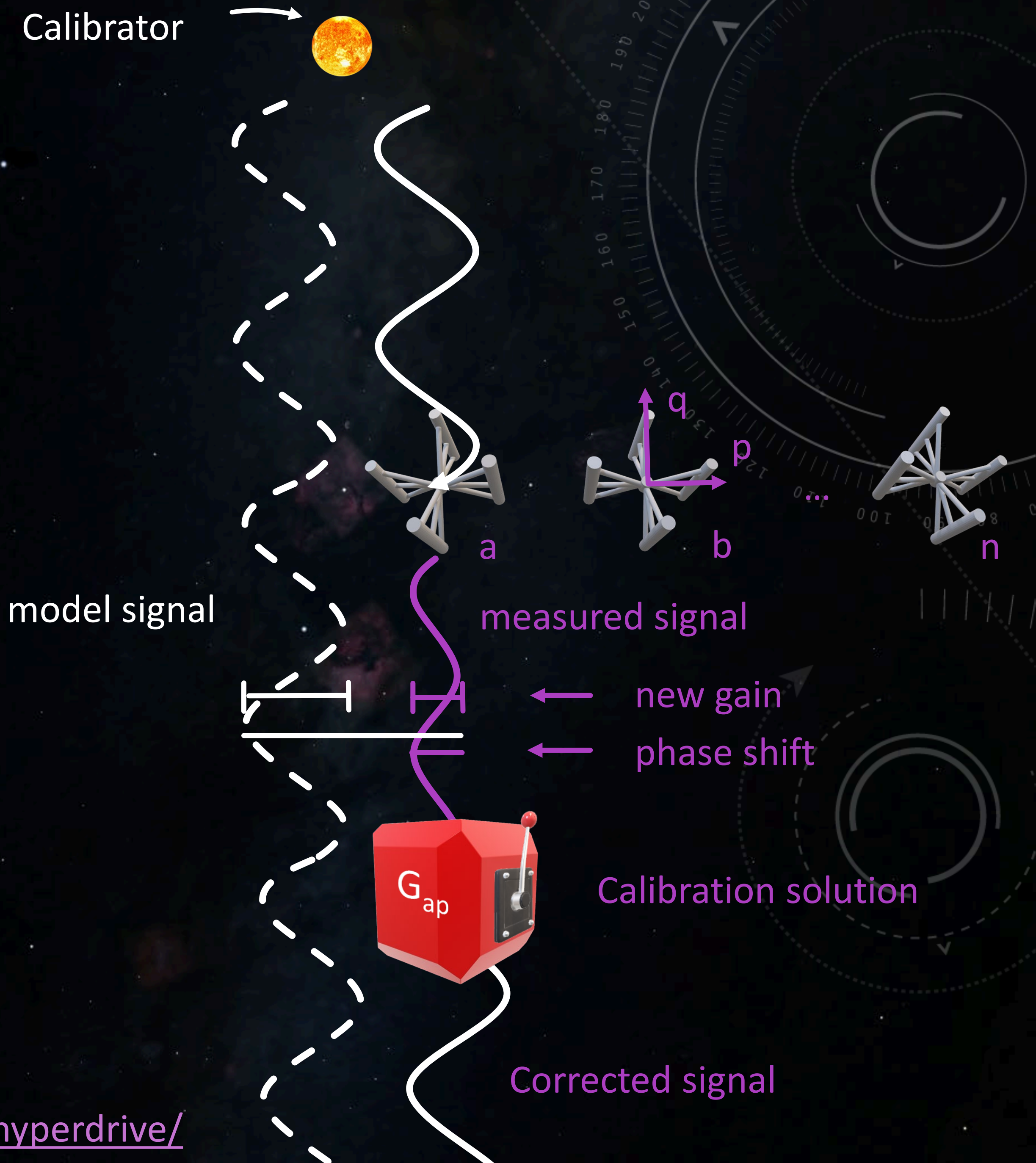
X. Cross-talk between antennas or tiles

Some effects can vary over time and/or frequency in ways that are difficult to model.

Calibration captures modelling errors to improve coherency

DIRECTION INDEPENDENT CALIBRATION

- Observe one or more **calibrators**: bright source with a known, stable brightness / polarimetry
- Synthesize calibrator visibilities, modelling as many effects as possible
- Use least-squares to solve for the gains (G) and phase shifts (ϕ) for each antenna (a,b,…), polarization (p,q), frequency and timeblock that minimizes the difference between model and measured visibilities



Hyperdrive documentation: https://mwatelescope.github.io/mwa_hyperdrive/

CALIBRATION CHALLENGE: GALACTIC CENTER

Hyperdrive vs MWA-Reduce/Calibrate vs RTS (DI only)

MWA Phase I, obsid 1089464560, ra=245.3°, dec= -26.7 °

1000 sources from LoBES catalog (no galactic center)

Minimum uvw cutoff: 50λ (97.187m)

50 iterations

Convergence thresholds: 1e-4, 1e-8

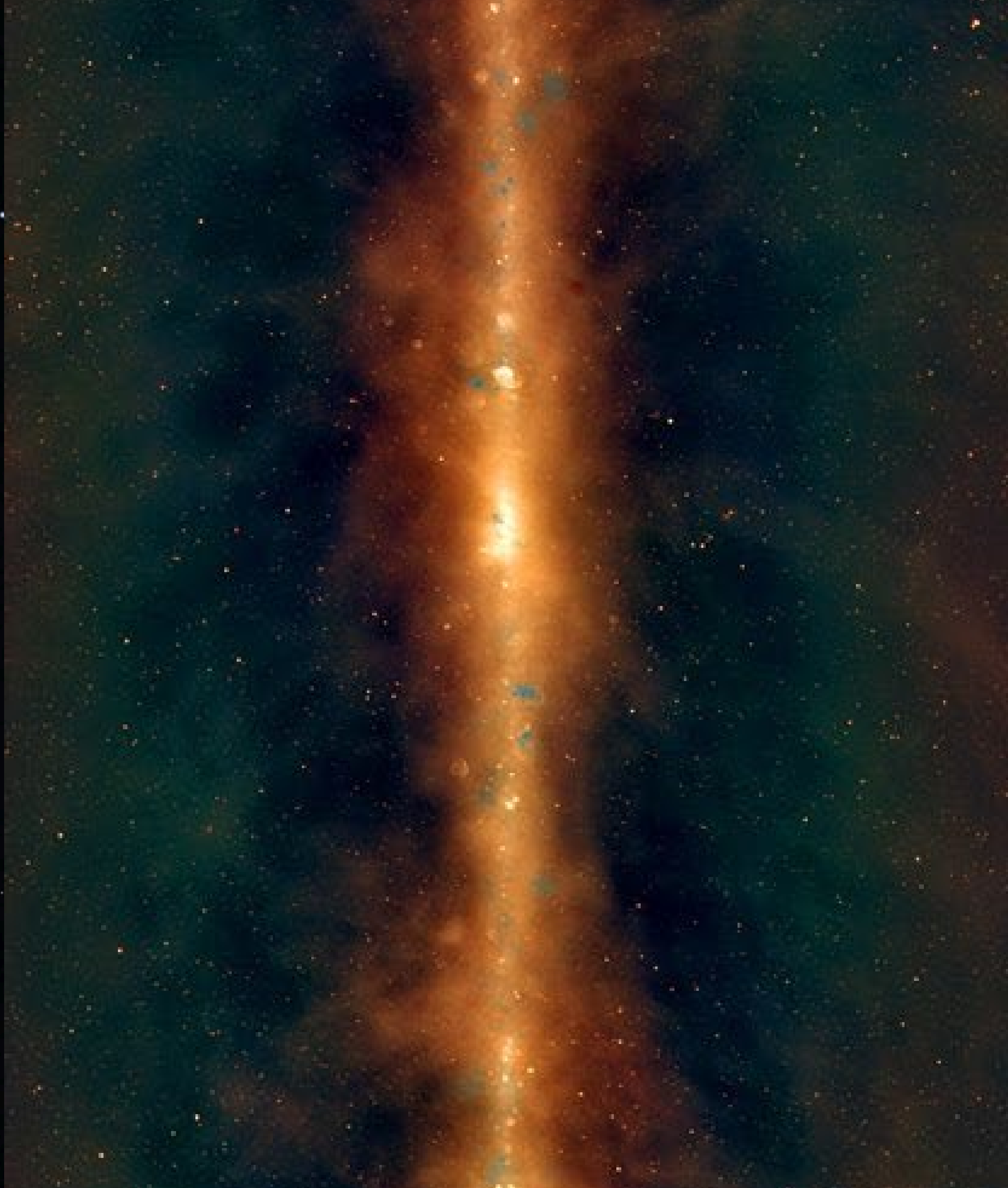


Image Credit: Dr. Natasha Hurley-Walker / GLEAM

CALIBRATION SOLUTION PHASE PLOTS

Mostly for diagnostic purposes

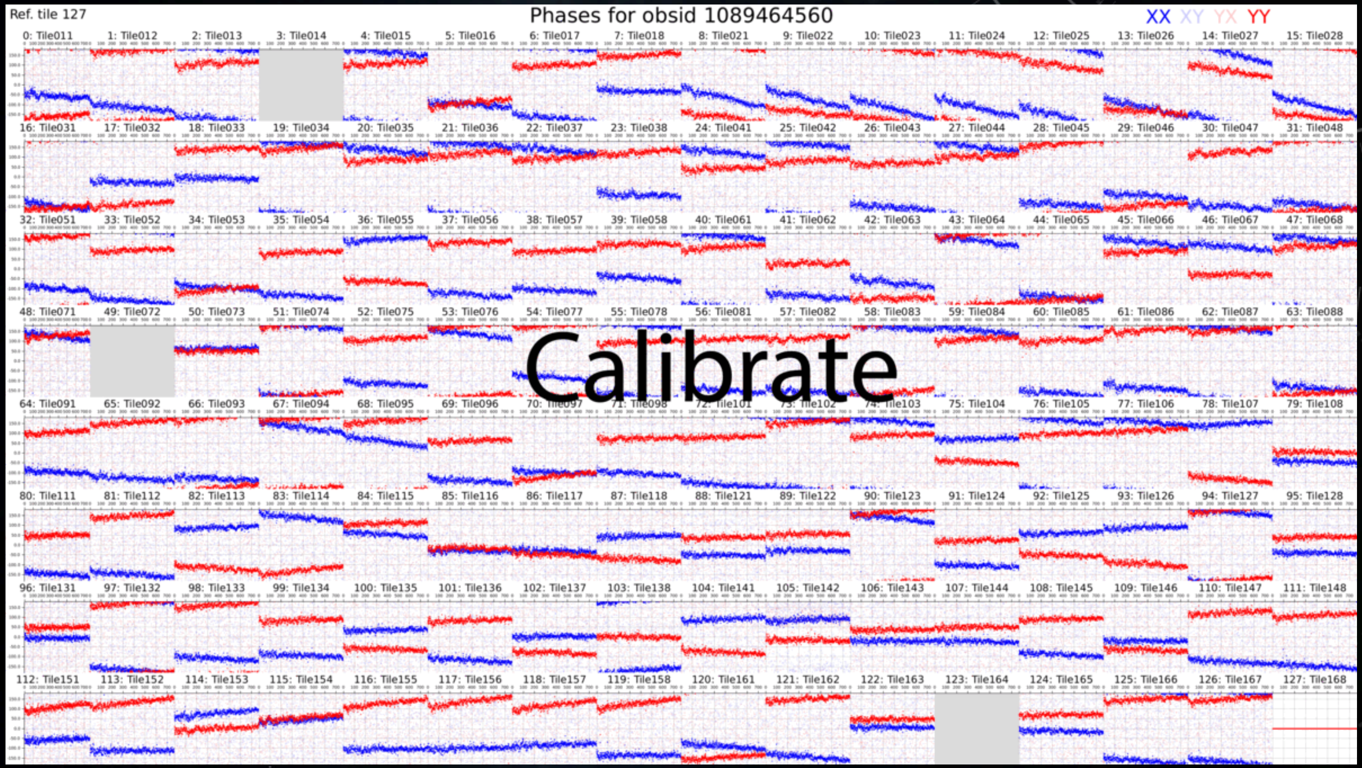
Plot phases and over frequency for each antenna and polarization.

Pick an antenna to use as a reference (127)

Hyperdrive can create these plots for many different calibration solution formats

Nice flat phase ramps show that the calibration went well.

Red = XX, Blue = YY



PERFORMANCE

	Desktop	Garrawarla
Hyperdrive	00:09:10	00:06:15
Calibrate*	02:10:00	03:27:00
RTS	-	00:00:17**

Hardware:

Desktop: Ryzen 9 3900X (12 cores, 24 threads) with 128GB memory

Garrawarla: 32 cores, 350GB memory, NVMe storage

** RTS used 25 nodes, others used 1

Software:

Hyperdrive: 0.2.0-alpha.11

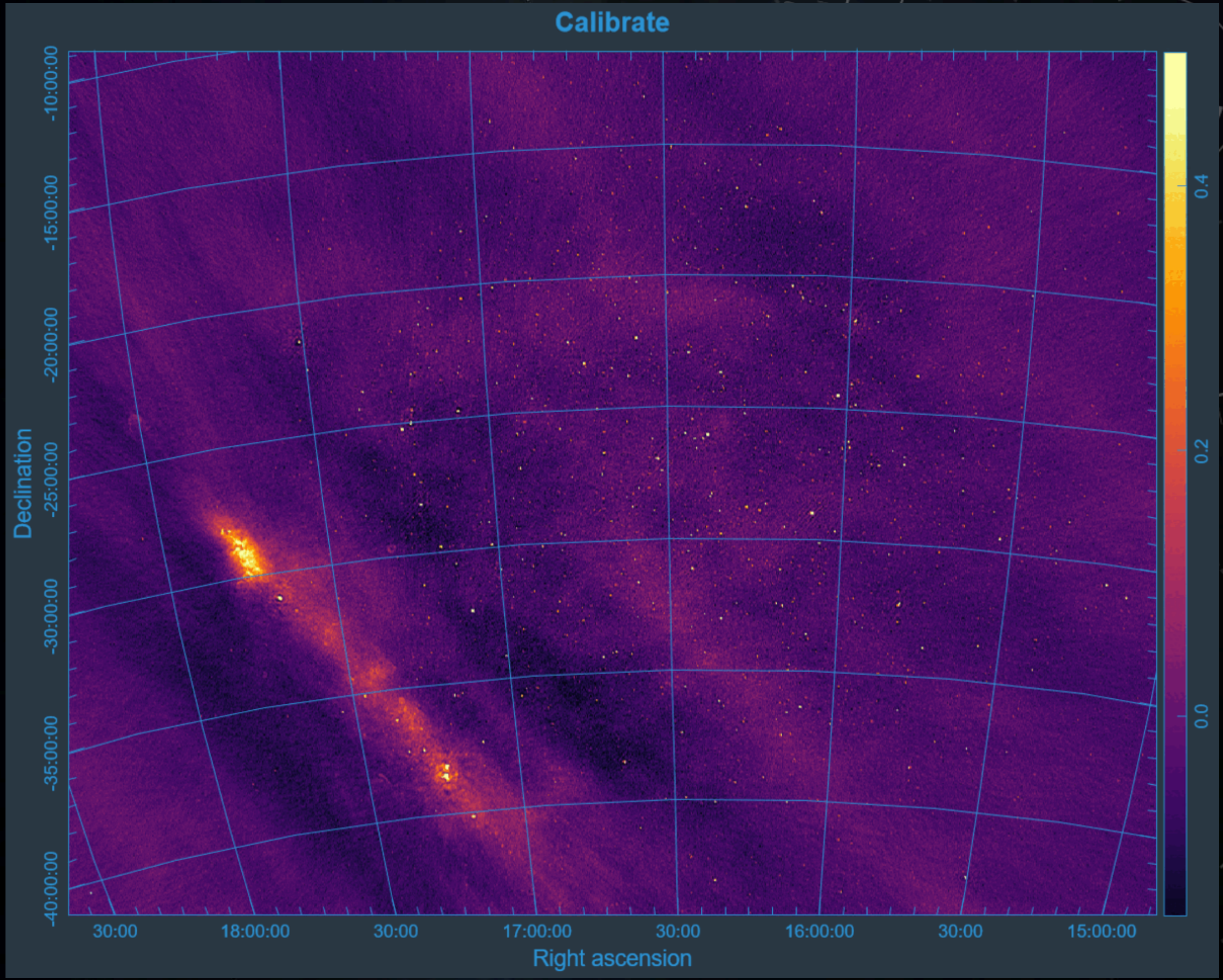
Mwa-reduce: git sha ef820c9 (no CUDA beam)

RTS: git sha c717651

IMAGES

Hyperdrive and Calibrate look good.

RTS… not so good.



DIRECTION DEPENDENT IONOSPHERIC CALIBRATION

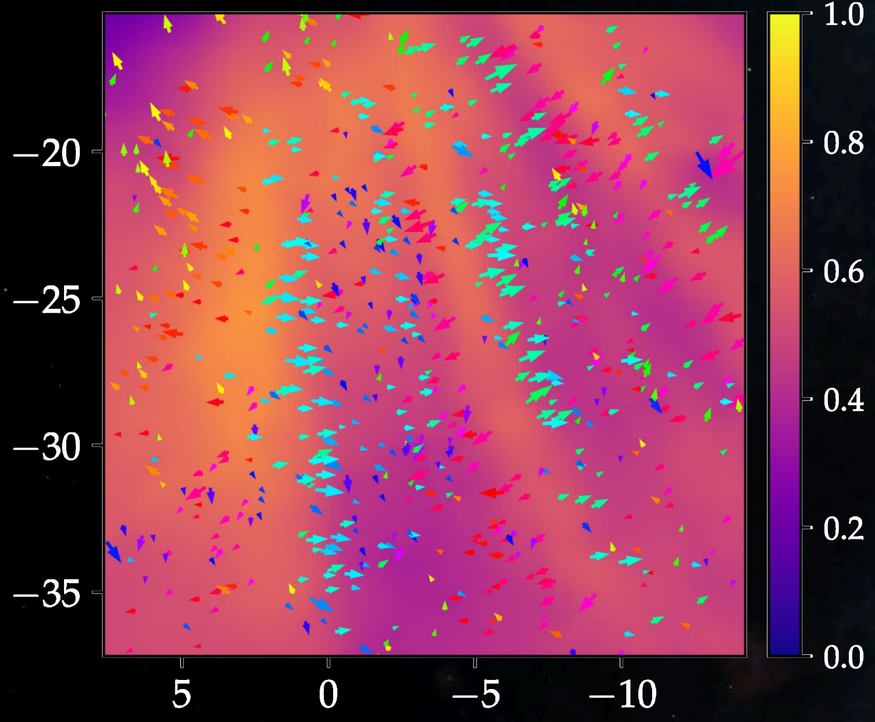
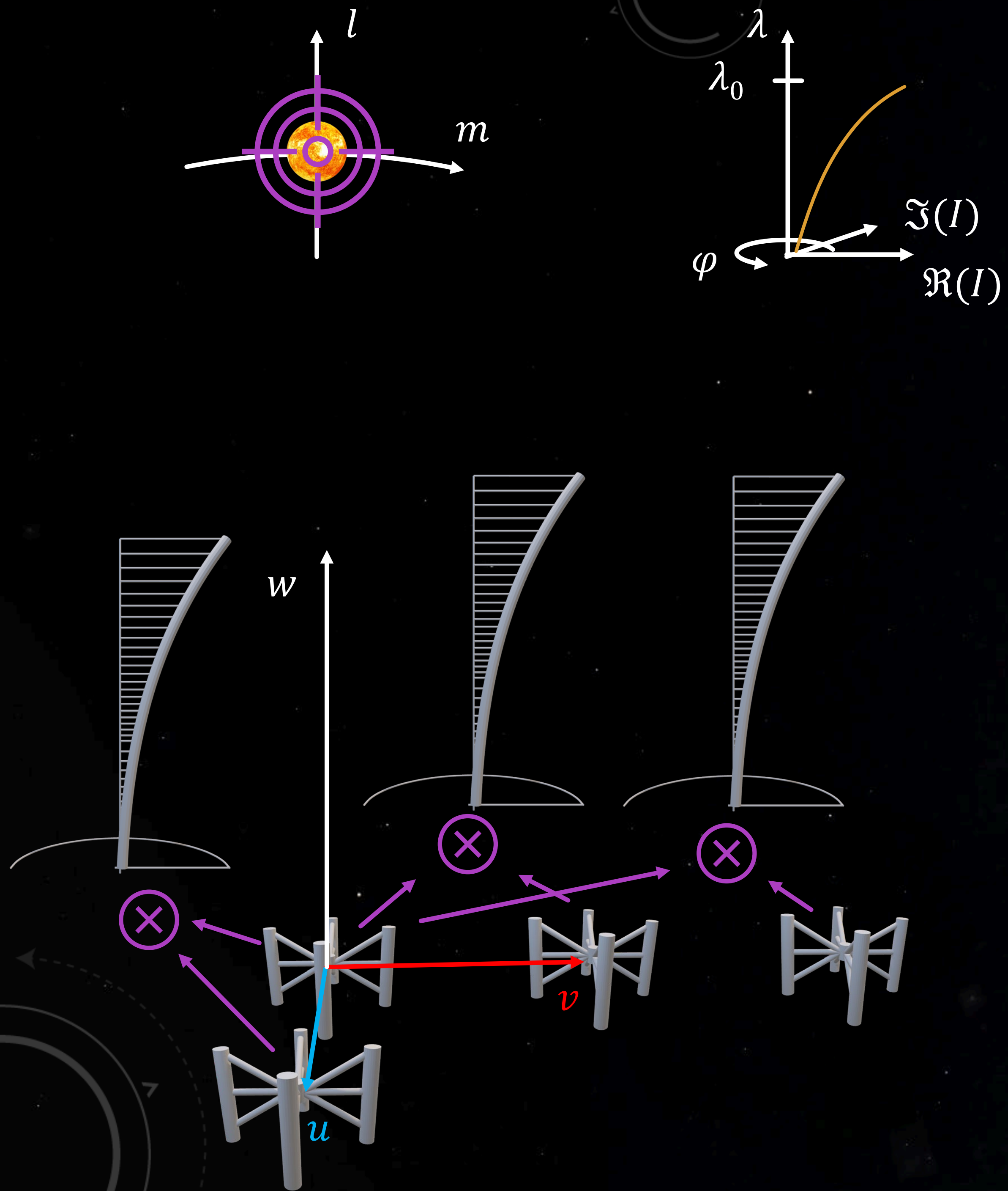


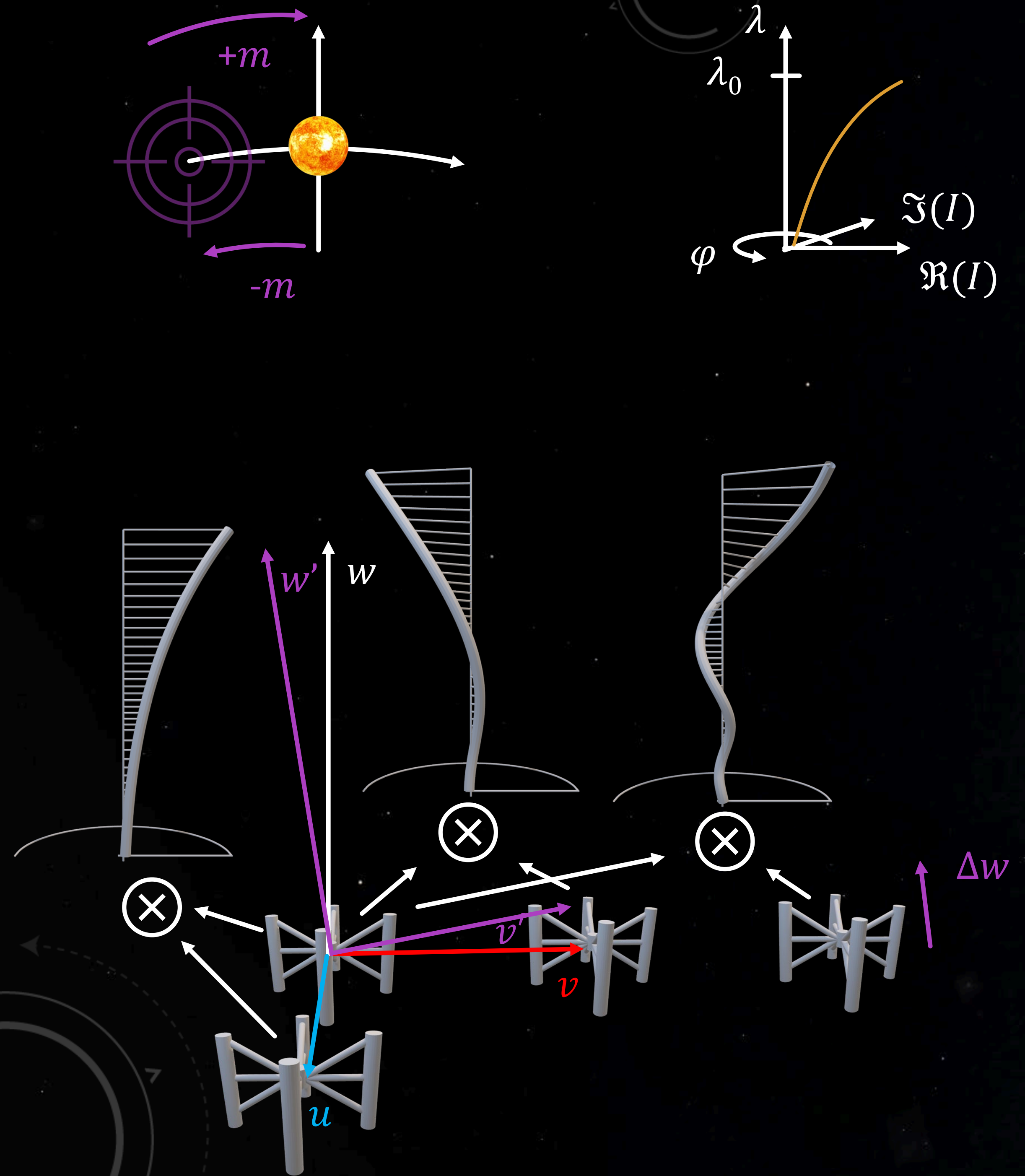
Image Credit: C. H. Jordan, Characterisation of the ionosphere above the Murchison Radio Observatory using the Murchison Widefield Array, MNRAS 2017

SYNTHESIZING INTERFEROMETER VISIBILITIES



- Only stokes I considered
- Single source at phase center ($l = 0, m = 0$) with $I = 1$ at λ_0 , negative spectral index
- Complex visibilities measured at baseline coordinates (u, v, w) is Fourier dual of power measured at sky coordinates (l, m, n) relative to phase center
- All baselines see the same power, visibilities have no imaginary component.

SOURCE POSITION OFFSETS



- When synthesizing visibilities offset from phase centre, shift phase centre in the opposite direction: $(-l, -m)$ before attenuating with beam
- Visibilities are rotated by φ, proportional to change in baseline's w-component:
  - $I' = e^{i\varphi} I$
  - $\varphi = 2\pi(\mathrm{w}' - \mathrm{w})\ \lambda$
- UVW coordinate of baseline also depend on wavelength
- Example: offset in m direction, only v baselines spiral with λ

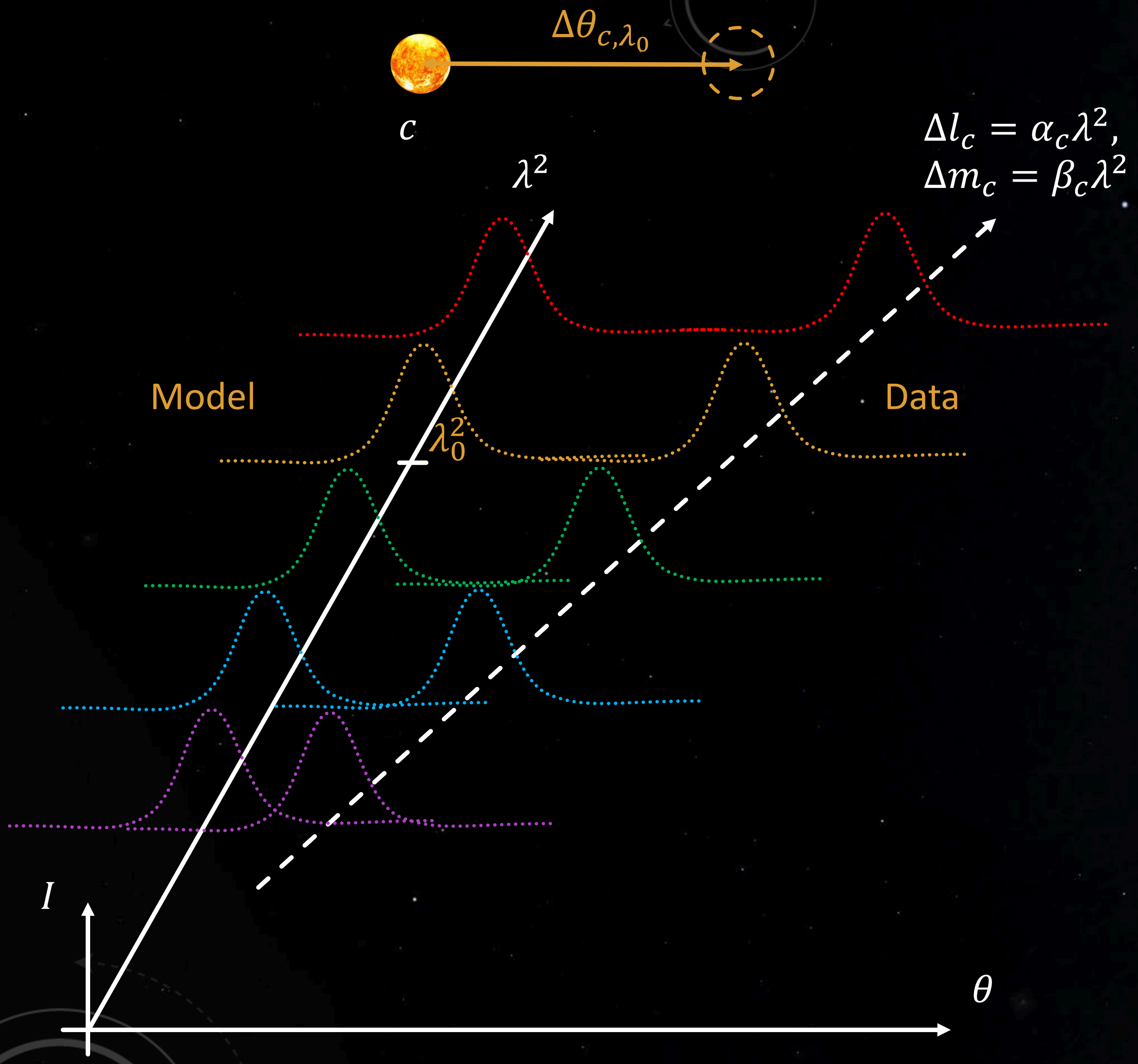
IONOSPHERIC EFFECTS

- Light travels slower through regions of higher electron density, is refracted by the ionosphere
- Causes a λ^2 depdendent position offset $\Delta\theta$ in (l, m):

$$\Delta\theta \simeq -\frac{1}{8\pi^2} \cdot \frac{e^2}{\epsilon_0 m_e} \cdot \lambda^2 \nabla_{\text{STEC}} \text{ [rad]}$$

  - e - electron charge
  - m_e - electron mass
  - ϵ_0 - vacuum permittivity
  - ∇_{STEC} - slant total electron content
- Direction of offset depends on STEC gradient seen by each source and baseline
- Cannot be corrected with direction independent calibration

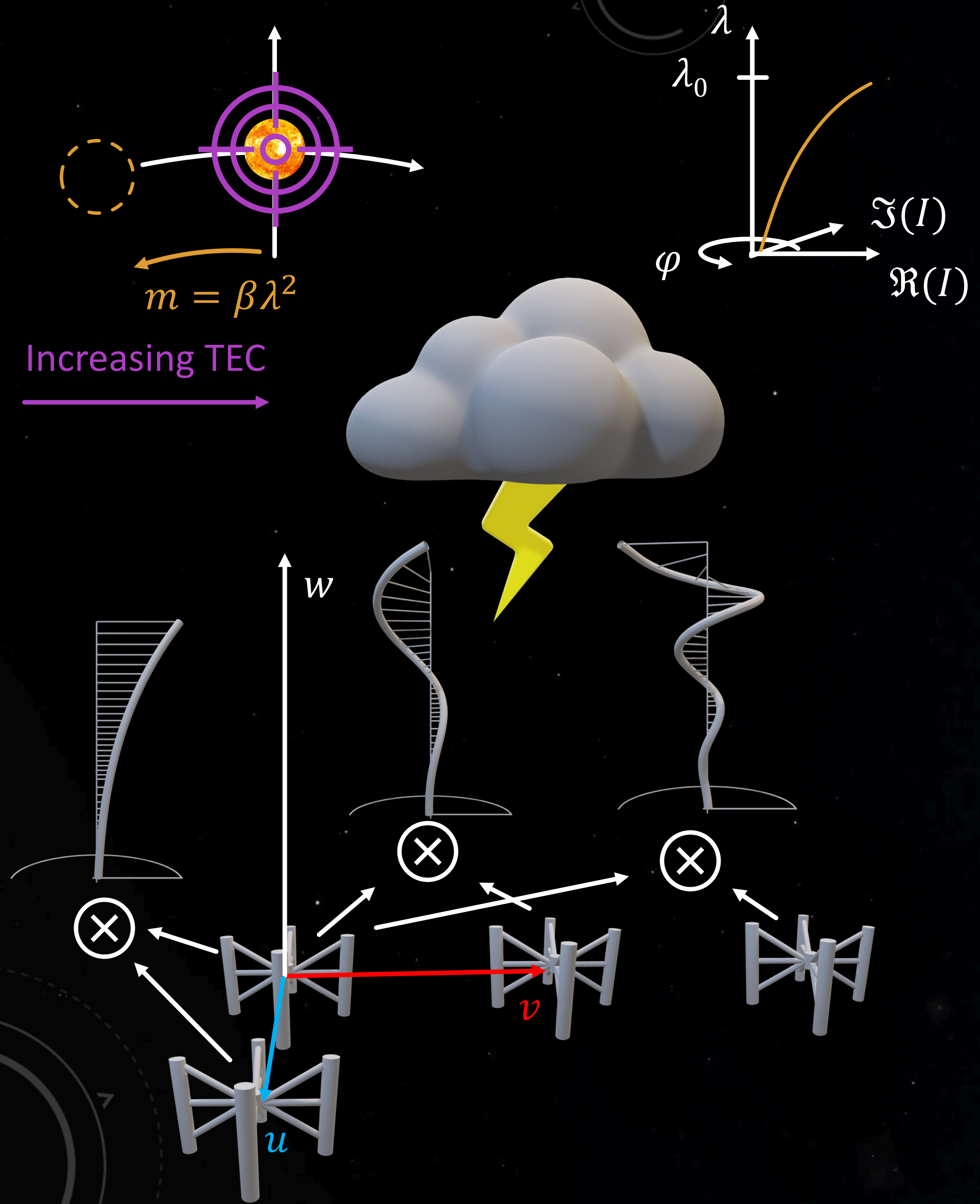
MODELLING IONOSPHERIC OFFSET



For a given calibrator, (c)

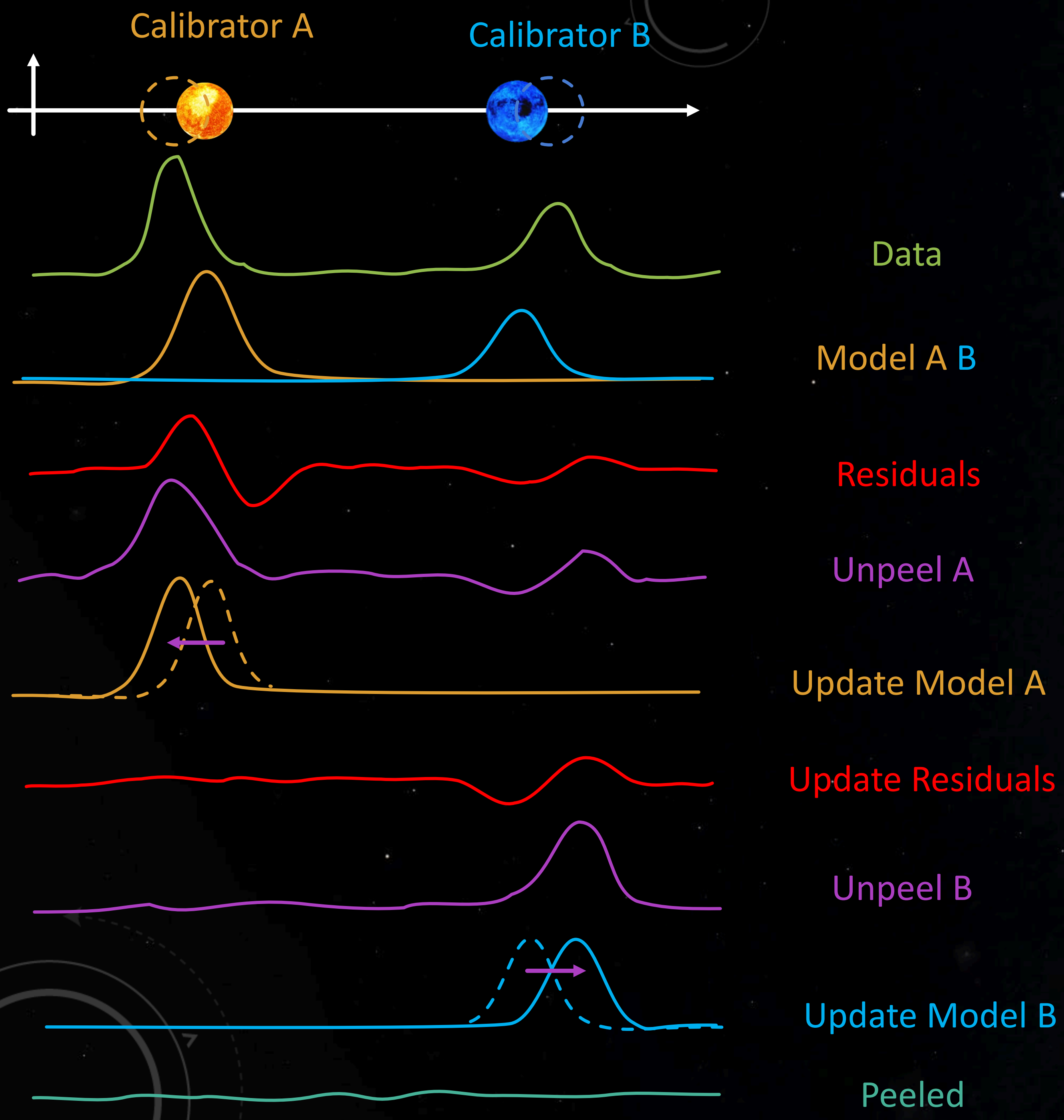
- Assume all baselines see the same ionospheric offset
- Fit constants of proportionality (α, β) for position offset (l, m) with λ^2

IONOSPHERIC POSITION OFFSETS



- Visibilities are rotated by φ, determined by component of baseline in the direction of TEC gradient:
  - $I' = e^{i\varphi} I$
  - $\varphi = 2\pi(\alpha u + \beta v)\,\lambda^2$
- UVW coordinate of each baseline depends on wavelength
- Example: TEC gradient aligned with m (β) causes an offset in m direction, only v baselines spiral with λ

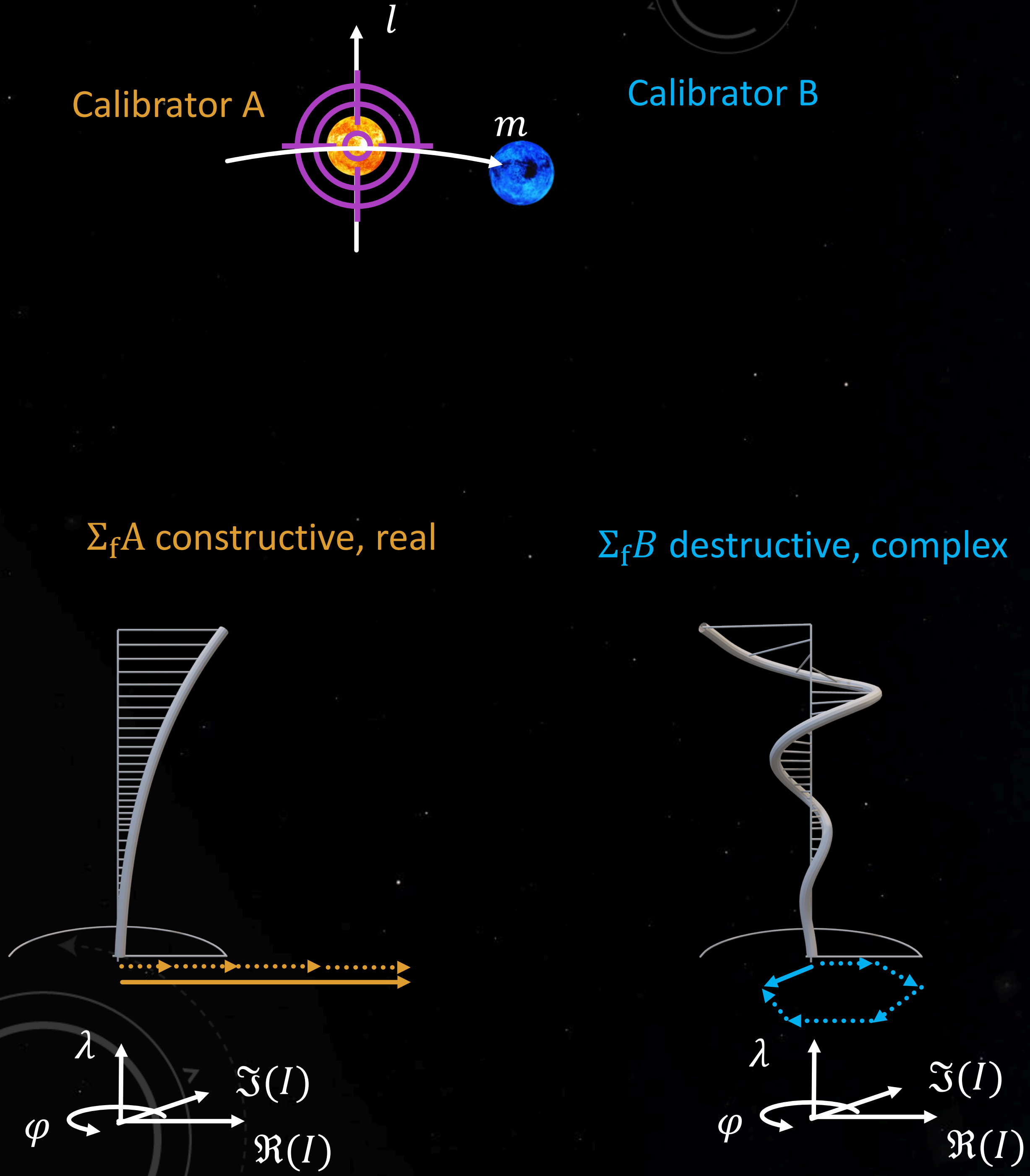
Position offset for comparison



PEELING OVERVIEW

- Iteratively improve a foreground model with information about ionospheric offsets by subtracting model from data
- Model is never perfect nor complete, so residuals remain
- For each calibrator, use unpeeled vis to update model
- For best results in model update, minimize residual power from other sources
- peel calibrators in decreasing order of brightness

FURTHER REDUCING RESIDUAL NOISE

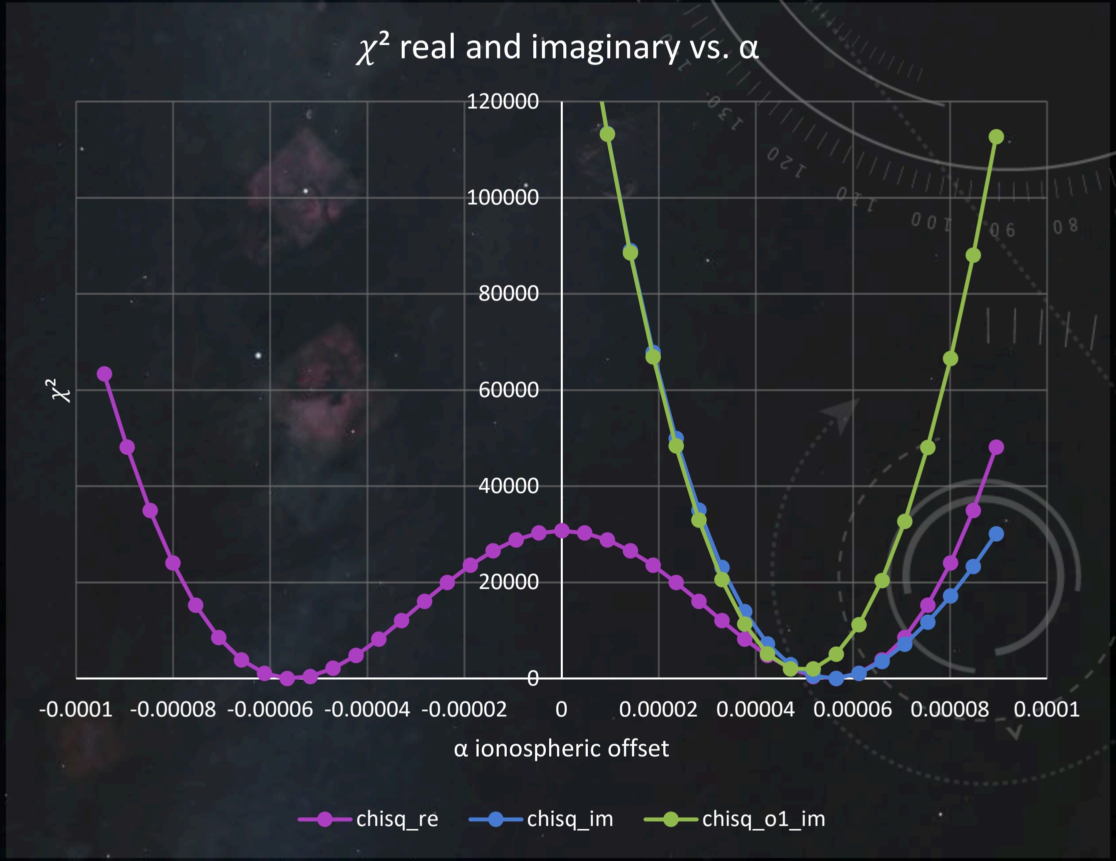


- Visibilities are phased to the calibrator and averaged over a frequency band
- sources near the phase center have visibilities added constructively because complex phases aligned
- sources offset from phase center have a phase shift over frequency, summing destructively
- Concentrating power at calibrator Improves signal-to-noise of unpeeled, peeling performance

COMPUTING IONOSPHERIC OFFSET

For each calibrator (c):

- Find $(\alpha_c + \beta_c)$ to minimize χ^2 , the least-squares error between rotated model and unpeeled
- Calibrator model is mostly real at phase center: $M^{(K)}_{jk,c,f} \sim \Re(M^{(K)}_{jk,c,f})$
- χ^2 has multiple minimums for real component, single minimum for imaginary component.
- For small offsets ($< 1'$), short baselines, $\alpha_c u_{jk,f} + \beta_c v_{jk,f} \ll 1$. Order 1 Taylor series approximation converges on real minimum with iteration
- Least squares fit on imaginary component:
- $\chi^2 \sim \sum_{jk,f} \left[\frac{\Im\left(U^{(K)}_{jk,c,f}\right) + 2\pi(\alpha_c u_{jk,f} + \beta_c v_{jk,f})\lambda^2 \Re(M^{(K)}_{jk,c,f})}{\sigma_{jk,f}}\right]^2$
- Solve for $\frac{\partial \chi^2}{\partial \alpha_c} = \frac{\partial \chi^2}{\partial \beta_c} = 0$



IONOSPHERIC POSITION FINDER

Suppose ionospheric phase shift $\delta\phi_{jk,c,f_0}$ perturbs source by $\delta l = \alpha_c \lambda_0^2$, $\delta m = \beta_c \lambda_0^2$

Averaged, unpeeled vis for calibrator $I'^{(K)}_{jk,c,f_0}$ should approximate phase shift of calibrator model I_c

$$I'^{(K)}_{jk,c,f_0} \approx I_c e^{-i\delta\phi_{jk,c,f_0}\lambda_0^2}$$

$$\approx I_c(1 - i\delta\phi_{jk,c,f_0}\lambda_0^2 + \cdots)$$

Taylor series expansion, Holds for $\delta\phi_{jk,c,f_0}\lambda_0^2 \ll 1$

Suppose model vis I_c has negligable imaginary component because it's at the phase centre

$$\Re\left(I'^{(K)}_{jk,c,f_0}\right) \approx I_c$$

$$\Im\left(I'^{(K)}_{jk,c,f_0}\right) \approx -2\pi I_c(\alpha_c u_{jk,f_0} + \beta_c v_{jk,f_0})\lambda_0^2$$

Least squares fit on imaginary component:

$$\chi^2 = \sum_{jk,f}\left[\frac{\Im\left(I'^{(K)}_{jk,c,f_0}\right) + 2\pi I_c(\alpha_c u_{jk,f_0} + \beta_c v_{jk,f_0})\lambda_0^2}{\sigma_{jk,f}}\right]^2$$

At $\frac{\partial\chi^2}{\partial\alpha_c} = \frac{\partial\chi^2}{\partial\beta_c} = 0$,

$$0 = \sum_{jk,f}\frac{\Im\left(I'^{(K)}_{jk,c,f_0}\right) + 2\pi I_c(\alpha_c u_{jk,f_0} + \beta_c v_{jk,f_0})\lambda_0^2}{\sigma_{jk,f}}\cdot\frac{u_{jk,f_0}\lambda_0^2}{\sigma_{jk,f}}$$

$$0 = \sum_{jk,f}\frac{\Im\left(I'^{(K)}_{jk,c,f_0}\right) + 2\pi I_c(\alpha_c u_{jk,f_0} + \beta_c v_{jk,f_0})\lambda_0^2}{\sigma_{jk,f}}\cdot\frac{v_{jk,f_0}\lambda_0^2}{\sigma_{jk,f}}$$

Rewrite with a terms defined as

$$a_{uv} = \sum_{jkf}\frac{u_{jk}v_{jk}\lambda_0^4}{\sigma_{jk,f}^2}, a_{uu} = \cdots, a_{vv} = \cdots$$

$$A_u = -\sum_{jkf}\frac{u_{jk}\Im\left(I'^{(K)}_{jk,c,f_0}\right)\lambda_0^2}{\sigma_{jkf}^2}, A_v = \cdots$$

$$A_u = 2\pi I_c(\alpha_c a_{uu} + \beta_c a_{uv})$$
$$A_v = 2\pi I_c(\alpha_c a_{uv} + \beta_c a_{vv})$$

Find relationship between α, β

$$2\pi I_c = \frac{A_u}{\alpha_c a_{uu} + \beta_c a_{uv}} = \frac{A_v}{\alpha_c a_{uv} + \beta_c a_{vv}}$$

$$A_u\alpha_c a_{uv} + A_u\beta_c a_{vv} = A_v\alpha_c a_{uu} + A_v\beta_c a_{uv}$$

$$\alpha_c(a_{uv}A_u - a_{uu}A_v) = \beta_c(a_{uv}A_v - a_{vv}A_u)$$

$$\alpha_c = \frac{\beta_c(a_{uv}A_v - a_{vv}A_u)}{a_{uv}A_u - a_{uu}A_v} \text{ and } \beta_c = \frac{\alpha_c(a_{uv}A_u - a_{uu}A_v)}{(a_{uv}A_v - a_{vv}A_u)}$$

Substitute α_c in to $A_u = 2\pi I_c(\alpha_c a_{uu} + \beta_c a_{uv})$?

$$A_u = 2\pi I_c\left(\frac{(a_{uv}A_v - a_{vv}A_u)}{(a_{uv}A_u - a_{uu}A_v)}a_{uu} + a_{uv}\right)\beta_c$$

$$A_u(a_{uv}A_u - a_{uu}A_v) = 2\pi I_c\big((a_{uv}A_v - a_{vv}A_u)a_{uu} + a_{uv}(a_{uv}A_u - a_{uu}A_v)\big)\beta_c$$

$$a_{uv}A_u - a_{uu}A_v = 2\pi I_c(-a_{uu}a_{vv} + a_{uv}^2)\beta_c$$

Substitute β_c in to $A_u = 2\pi I_c(\alpha_c a_{uu} + \beta_c a_{uv})$

$$A_u = 2\pi I_c\left(\alpha_c a_{uu} + \frac{\alpha_c(a_{uv}A_u - a_{uu}A_v)}{(a_{uv}A_v - a_{vv}A_u)}a_{uv}\right)$$

$$A_u(a_{uv}A_v - a_{vv}A_u) = 2\pi I_c(a_{uu}(a_{uv}A_v - a_{vv}A_u) + (a_{uv}A_u - a_{uu}A_v)a_{uv})\alpha_c$$

$$a_{uv}A_v - a_{vv}A_u = 2\pi I_c(-a_{uu}a_{vv} + a_{uv}^2)\alpha_c$$

Define $\Delta = a_{uu}a_{vv} - a_{uv}^2$

$$\alpha_c = \frac{a_{vv}A_u - a_{uv}A_v}{2\pi I_c\Delta}$$

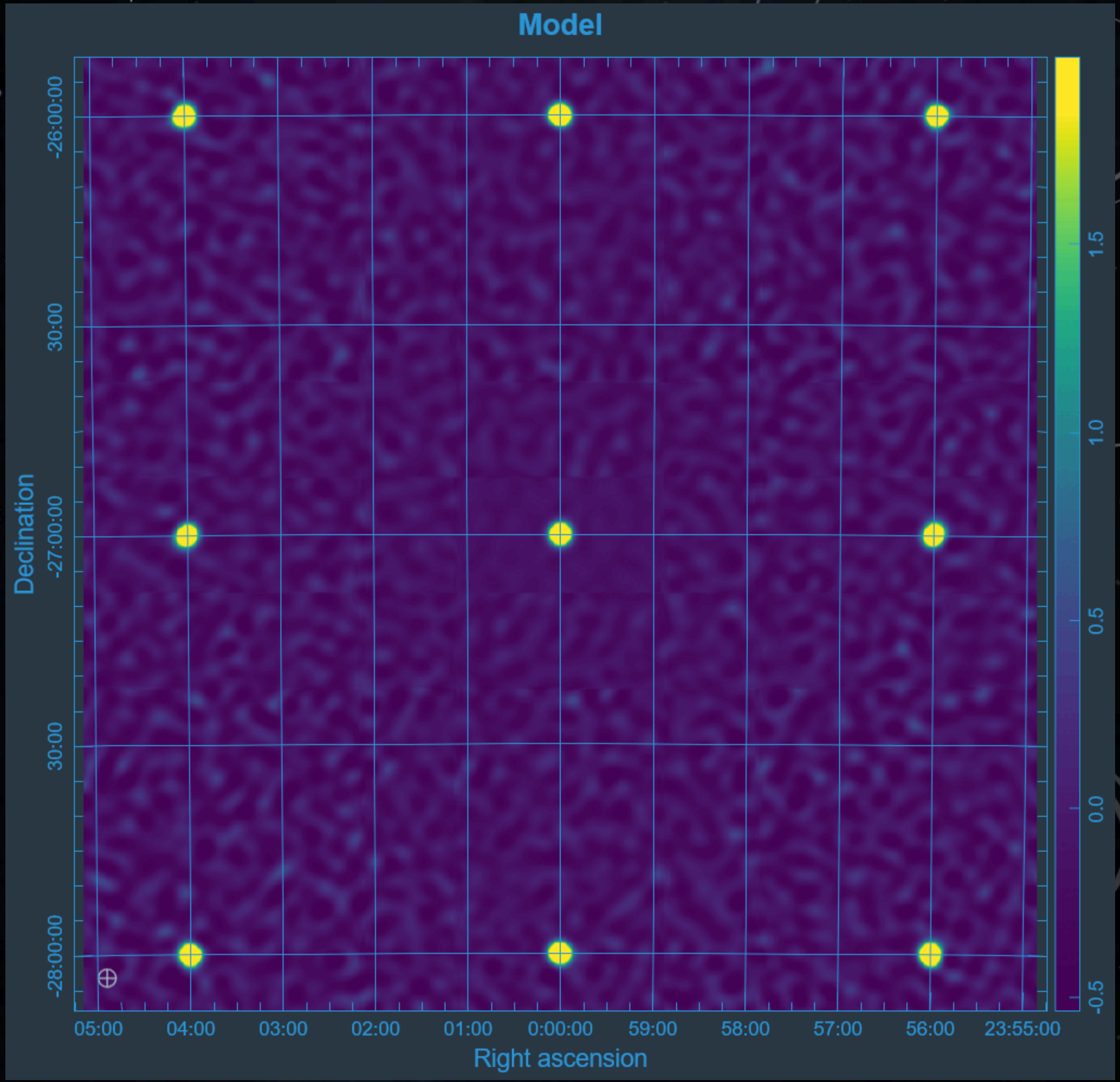
$$\beta_c = \frac{a_{uu}A_v - a_{uv}A_u}{2\pi I_c\Delta}$$

Derived from Mitchel Et Al (2008) “Real Time Calibration Of The Murchison Widefield Array”

Lost memo: R. J. Sault, “Visibility-based position finder,” MWA-LFD Memo in preparation, 2007

STATUS

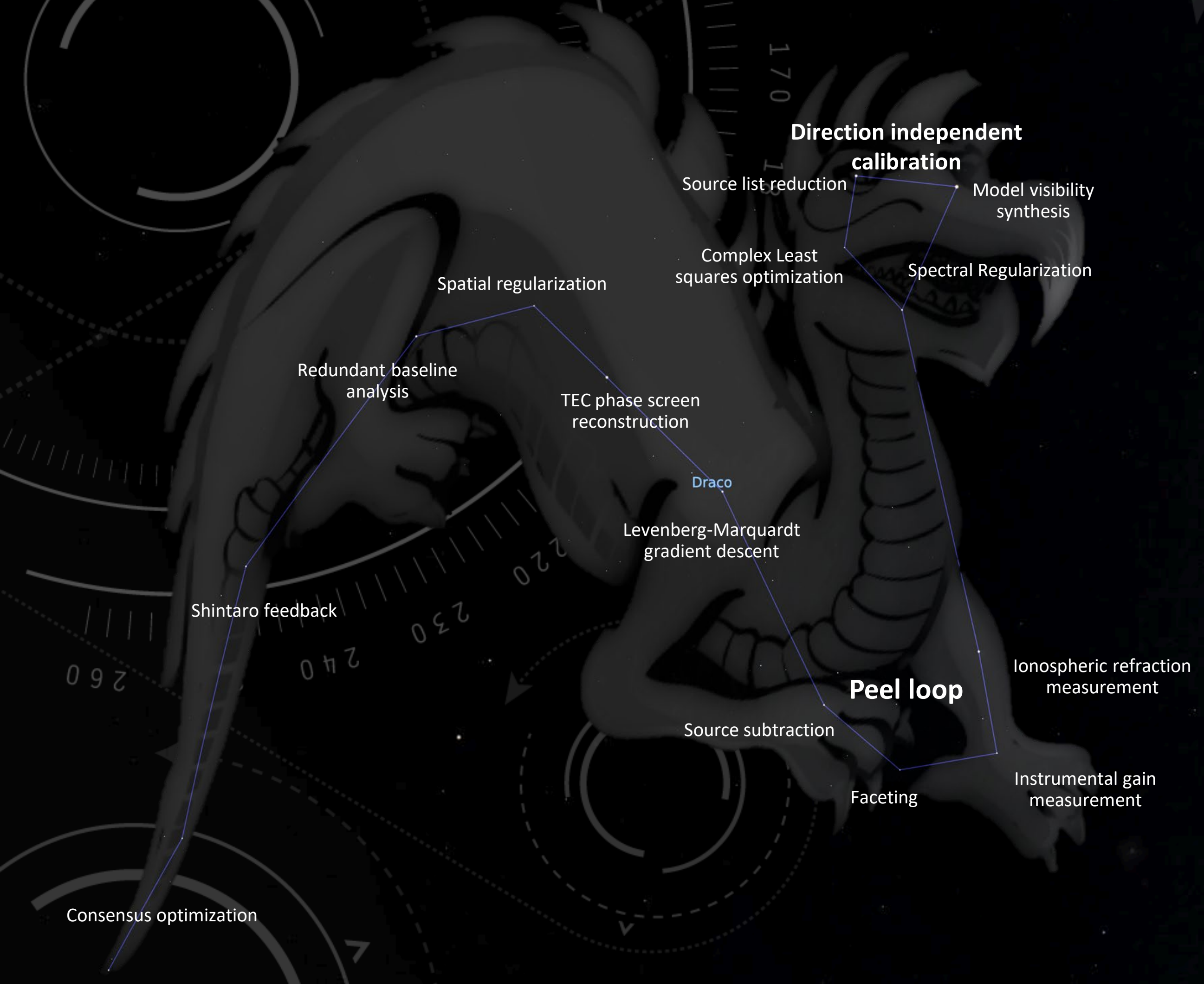
- RTS reverse engineering took longer than expected
  - Missing documentation
  - Code optimized for HPC performance, not readability
  - Difficulties running RTS on synthesized visibilities
- Initial RTS-based peeling implementation works with synthesized sample data
  - Needs to be scaled up to real data



FUTURE WORK

- Fully Implement DD calibration in hyperdrive
- Validate against other implementations:
  - RTS
  - yandasoft
- Investigate advanced calibration ideas:
  - Levenberg-Marquardt gradient descent
  - Faceting
  - Spectral regularization
  - Redundant baseline analysis
  - TEC phase screen reconstruction

THANK YOU



Slides link:
tinyurl.com/2xtpmkz8

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